

Trigonometry

Trigonometry is one of the important branches in the history of mathematics and this concept is given by a Greek mathematician Hipparchus. Here, we will study the relationship between the sides and angles of a right-angled triangle. The basics of trigonometry define three primary functions which are sine, cosine and tangent.

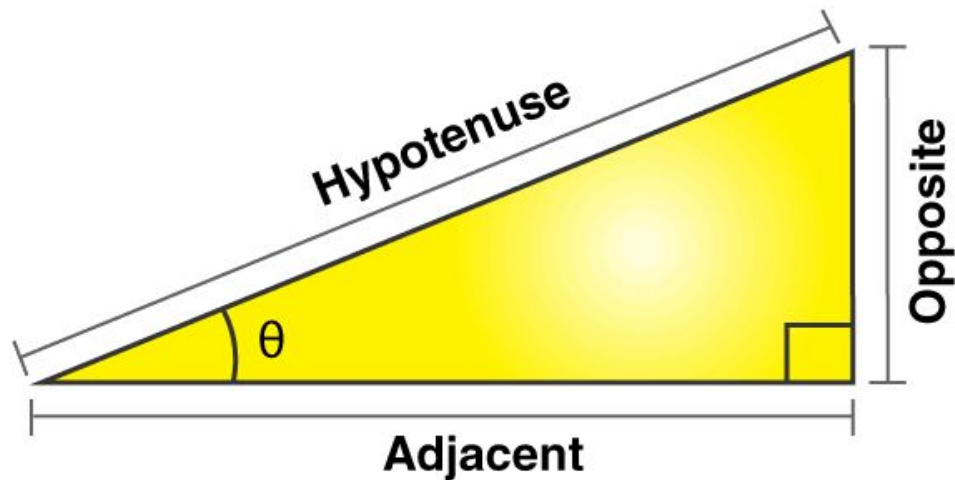
Trigonometry is one of those divisions in mathematics that helps in finding the angles and missing sides of a triangle with the help of trigonometric ratios. The angles are either measured in radians or degrees. The commonly used trigonometry angles are 0° , 30° , 45° , 60° and 90° .

Trigonometry can be divided into two sub-branches called plane trigonometry and spherical geometry. Here, you will learn about the trigonometric formulas, functions, and ratios, etc.

Trigonometry Ratios-Sine, Cosine, Tangent

The trigonometric ratios of a triangle are also called the trigonometric functions. Sine, cosine, and tangent are 3 important trigonometric functions and are abbreviated as sin, cos, and tan. Let us see how are these ratios or functions, evaluated in case of a right-angled triangle.

Consider a right-angled triangle, where the longest side is called the hypotenuse, and the sides opposite to the hypotenuse are referred to as the adjacent and opposite sides.



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Six Important Trigonometric Functions

The six important trigonometric functions (trigonometric ratios) are calculated using the below formulas and considering the above figure. It is necessary to get knowledge about the sides of the right triangle because it defines the set of important trigonometric functions.

Functions	Abbreviation	Relationship to sides of a right triangle
Sine Function	sin	Opposite side/ Hypotenuse
Tangent Function	tan	Opposite side / Adjacent side
Cosine Function	cos	Adjacent side / Hypotenuse

Cosecant Function	cosec	Hypotenuse / Opposite side
Secant Function	sec	Hypotenuse / Adjacent side
Cotangent Function	cot	Adjacent side / Opposite side

Trigonometry Angles

The trigonometry angles which are commonly used in trigonometry problems are 0° , 30° , 45° , 60° and 90° . The trigonometric ratios such as sine, cosine and tangent of these angles are easy to memorize. We will also show the table where all the ratios and their respective angle's values are mentioned. To find these angles we have to draw a right-angled triangle, in which one of the acute angles will be the corresponding trigonometry angle. These angles will be defined with respect to the ratio associated with it.

For example, in a right-angled triangle,

$$\sin \theta = \text{Perpendicular/Hypotenuse}$$

$$\text{or } \theta = \sin^{-1} (P/H)$$

Similarly,

$$\theta = \cos^{-1} (\text{Base/Hypotenuse})$$

$$\theta = \tan^{-1} (\text{Perpendicular/Base})$$

Trigonometry Table

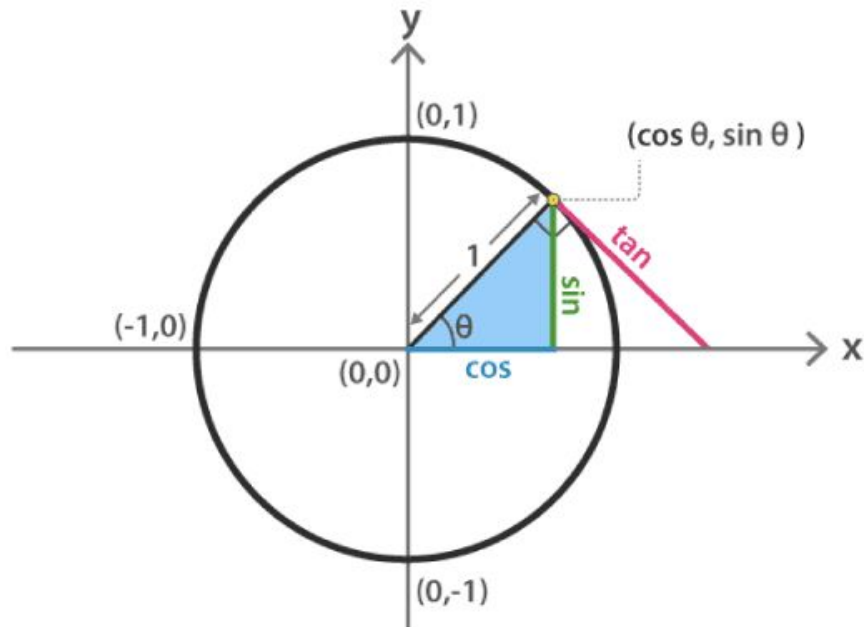
Check the table for common angles which are used to solve many trigonometric problems involving trigonometric ratios.

Angles	0°	30°	45°	60°	90°
Sin θ	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
Cos θ	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
Tan θ	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	∞
Cosec θ	∞	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1
Sec θ	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	∞
Cot θ	∞	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0

In the same way, we can find the trigonometric ratio values for angles beyond 90 degrees, such as 180°, 270° and 360°.

Unit Circle

The concept of unit circle helps us to measure the angles of cos, sin and tan directly since the centre of the circle is located at the origin and radius is 1. Consider theta be an angle then,



Suppose the length of the perpendicular is y and of base is x . The length of the hypotenuse is equal to the radius of the unit circle, which is 1.

Therefore, we can write the trigonometry ratios as;

Sin θ	$y/1 = y$
Cos θ	$x/1 = x$
Tan θ	y/x

List of Trigonometry Formulas

The Trigonometric formulas or Identities are the equations which are true in the case of Right-Angled Triangles. Some of the special trigonometric identities are given below –

1. Pythagorean Identities

- $\sin^2\theta + \cos^2\theta = 1$
- $\tan^2\theta + 1 = \sec^2\theta$

- $\cot 2\theta + 1 = \operatorname{cosec} 2\theta$
- $\sin 2\theta = 2 \sin \theta \cos \theta$
- $\cos 2\theta = \cos^2\theta - \sin^2\theta$
- $\tan 2\theta = 2 \tan \theta / (1 - \tan^2\theta)$
- $\cot 2\theta = (\cot^2\theta - 1) / 2 \cot \theta$

2. Sum and Difference identities-

For angles u and v , we have the following relationships:

- $\sin(u + v) = \sin(u)\cos(v) + \cos(u)\sin(v)$
- $\cos(u + v) = \cos(u)\cos(v) - \sin(u)\sin(v)$
- $\tan(u+v) =$
- $\frac{\tan(u) + \tan(v)}{1 - \tan(u) \tan(v)}$
- $\frac{\tan(u) - \tan(v)}{1 + \tan(u) \tan(v)}$
- $\sin(u - v) = \sin(u)\cos(v) - \cos(u)\sin(v)$
- $\cos(u - v) = \cos(u)\cos(v) + \sin(u)\sin(v)$
- $\tan(u-v) =$
- $\frac{\tan(u) - \tan(v)}{1 + \tan(u) \tan(v)}$

3. If **A, B and C** are angles and **a, b and c** are the sides of a triangle, then,

Sine Laws

- $a/\sin A = b/\sin B = c/\sin C$

Cosine Laws

- $c^2 = a^2 + b^2 - 2ab \cos C$
- $a^2 = b^2 + c^2 - 2bc \cos A$
- $b^2 = a^2 + c^2 - 2ac \cos B$

Trigonometry Identities

The three important trigonometric identities are:

- $\sin^2\theta + \cos^2\theta = 1$
- $\tan^2\theta + 1 = \sec^2\theta$
- $\cot^2\theta + 1 = \operatorname{cosec}^2\theta$

Euler's Formula for trigonometry

As per the euler's formula,

$$e^{ix} = \cos x + i \sin x$$

Where x is the angle and i is the imaginary number.

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$$\sin^2\theta + \cos^2\theta = 1$$

$$\tan^2\theta + 1 = \sec^2\theta$$

$$\cot^2\theta + 1 = \operatorname{cosec}^2\theta$$

Euler's Formula for trigonometry

As per the euler's formula,

$$e^{ix} = \cos x + i \sin x$$

Where x is the angle and i is the imaginary number.

$$\sin x = \frac{e^{ix} - e^{-ix}}{2i} \quad \cos x = \frac{e^{ix} + e^{-ix}}{2} \quad \tan x = \frac{e^{ix} - e^{-ix}}{e^{ix} + e^{-ix}}$$

Trigonometry Basics

The three basic functions in trigonometry are sine, cosine and tangent. Based on these three functions the other three functions that are cotangent, secant and cosecant are derived.

All the trigonometrical concepts are based on these functions. Hence, to understand trigonometry further we need to learn these functions and their respective formulas at first.

If θ is the angle in a right-angled triangle, then

$$\sin \theta = \text{Perpendicular/Hypotenuse}$$

$$\cos \theta = \text{Base/Hypotenuse}$$

$$\tan \theta = \text{Perpendicular/Base}$$

Perpendicular is the side opposite to the angle θ .

The base is the adjacent side to the angle θ .

The hypotenuse is the side opposite to the right angle

The other three functions i.e. cot, sec and cosec depend on tan, cos and sin respectively, such as:

$$\cot \theta = 1/\tan \theta$$

$$\sec \theta = 1/\cos \theta$$

$$\operatorname{cosec} \theta = 1/\sin \theta$$

Hence,

$$\cot \theta = \text{Base/Perpendicular}$$

$$\sec \theta = \text{Hypotenuse/Base}$$

$$\operatorname{cosec} \theta = \text{Hypotenuse/Perpendicular}$$

Trigonometry Examples

There are many real-life examples where trigonometry is used broadly.

If we have been given with height of the building and the angle formed when an object is seen from the top of the building, then the distance between object and bottom of the building can be determined by using the tangent function, such as \tan of angle is equal to the ratio of the height of the building and the distance. Let us say the angle is α , then

$$\tan \alpha = \text{Height/Distance between object \& building}$$

$$\text{Distance} = \text{Height}/\tan \alpha$$

Let us assume that height is 20m and the angle formed is 45 degrees, then

$$\text{Distance} = 20/\tan 45^\circ$$

$$\text{Since, } \tan 45^\circ = 1$$

$$\text{So, Distance} = 20 \text{ m}$$

Applications of Trigonometry

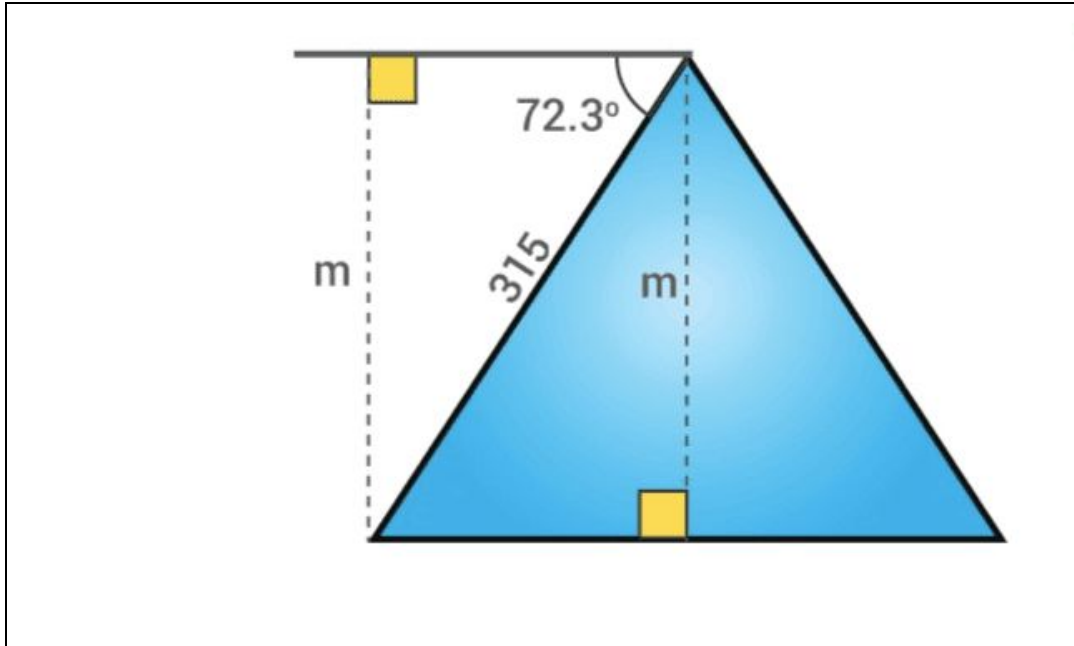
- Its applications are in various fields like oceanography, seismology, meteorology, physical sciences, astronomy, acoustics, navigation, electronics, etc.
- It is also helpful to measure the height of the mountain, find the distance of long rivers, etc.

Trigonometry Problems and Solutions

Example 1: Two friends, Rakesh and Vishal started climbing a pyramid-shaped hill. Rakesh climbs 315 m and finds that the angle of depression is 72.3 degrees from his starting point. How high he is from the ground.

Solution: Let m is the height above the ground.

To find: Value of m



To solve m , use the sine ratio.

$$\sin 72.3^\circ = m/315$$

$$0.953 = m/315$$

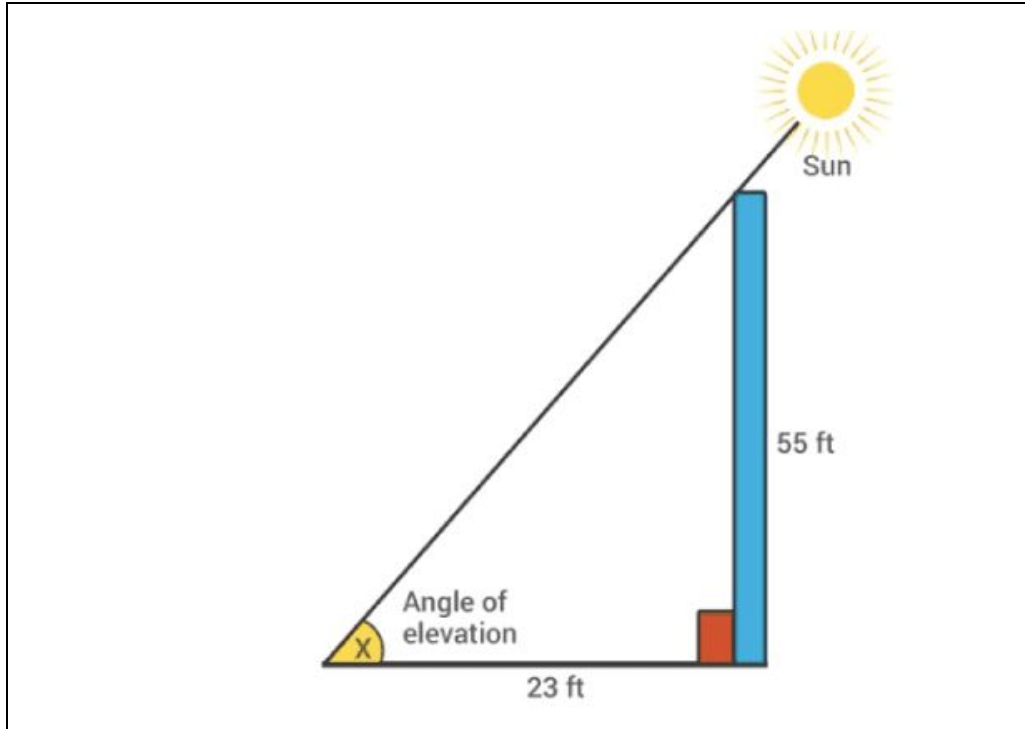
$$m = 315 \times 0.953$$

$$m = 300.195 \text{ mtr}$$

The man is 300.195 mtr above the ground.

Example 2: A man is observing a pole of height 55 foot. According to his measurement, pole cast a 23 feet long shadow. Can you help him to know the angle of elevation of the sun from the tip of shadow?

Solution:



Let x be the angle of elevation of the sun, then

$$\tan x = 55/23 = 2.391$$

$$x = \tan^{-1}(2.391)$$

$$\text{or } x = 67.30 \text{ degrees}$$

Trigonometry Questions

Practice these questions given here to get a deep knowledge of Trigonometry. Use the formulas and table given in this article wherever necessary.

Q.1: In $\triangle ABC$, right-angled at B, $AB=22$ cm and $BC=17$ cm. Find:

(a) $\sin A \cos B$

(b) $\tan A \tan B$

Q.2: If $12\cot \theta = 15$, then find $\sec \theta$.

Q.3: In ΔPQR , right-angled at Q, $PR + QR = 30$ cm and $PQ = 10$ cm. Determine the values of $\sin P$, $\cos P$ and $\tan P$.

Q.4: If $\sec 4\theta = \operatorname{cosec}(\theta - 300)$, where 4θ is an acute angle, find the value of A.

Frequently Asked Questions

What do you Mean by Trigonometry?

Trigonometry is one of the branches of mathematics which deals with the relationship between the sides of a triangle (right triangle) with its angles. There are 6 trigonometric functions for which the relation between sides and angles are defined. Learn more all about trigonometry now by visiting BYJU'S.

What are the six basic Trigonometric Functions?

There are 6 trigonometric functions which are:

- Sine function
- Cosine function
- Tan function
- Sec function
- Cot function
- Cosec function

What is the formula for six trigonometry functions?

The formula for six trigonometry functions are:

Sine A = Opposite side/Hypotenuse

Cos A = Adjacent side / Hypotenuse

Tan A = Opposite side / Adjacent side

Cot A = Adjacent side / Opposite side

Sec A = Hypotenuse / Adjacent side

Cosec A = Hypotenuse / Opposite side

What is the primary function of trigonometry?

The three primary functions of trigonometry are Sine function, Cosine function and Tangent Function.

Who is the founder of trigonometry?

A greek astronomer, geographer and mathematician, Hipparchus discovered the concept of trigonometry.

What are the Applications of Trigonometry in Real Life?

One of the most important real-life applications of trigonometry is in the calculation of height and distance. Some of the sectors where the concepts of trigonometry are extensively used are aviation department, navigation, criminology, marine biology, etc. Learn more about the applications of trigonometry here.

**Q : What is the value of
is:**

- (A) 3
- (B) 2
- (C) 1
- (D) 0

Answer

Correct Answer : C

Q : A tower standing on a horizontal plane subtends a certain angle at a point 160 m apart from the foot of the tower. On advancing 100 m towards it, the tower is found to subtend an angle twice as before. The height of the tower is

- (A) 80 m
- (B) 100 m

(C) 160 m

(D) 200 m

Answer

Correct Answer : A

Q : If $\sin\theta - \cos\theta = 7/13$ and $00 < \theta < 900$, then the value of $\sin\theta + \cos\theta$ is :

(A) 17/13

(B) 13/17

(C) 1/13

(D) 1/17

Answer

Correct Answer : A

Q : What is the value of

$$[\tan^2(90 - \theta) - \sin^2(90 - \theta)]\operatorname{cosec}^2(90 - \theta)\cot^2(90 - \theta)?$$

(A) 0

(B) 1

(C) -1

(D) 2

Answer

Correct Answer : B

$$1 + \frac{\tan^2 A}{1 + \sec A} ?$$

Q : What is the value of the following

(A) cosec A

(B) cos A

(C) secA

(D) sinA

Answer

Correct Answer : C

Q : If $\tan 2\theta + \cot 2\theta = 2$, then what is the value of $2\sec\theta \operatorname{cosec}\theta$?

(A) 0

(B) 1

(C) 2

(D) 4

Answer

Correct Answer : D

1. If $\sec \alpha = \frac{5}{4}$, then $\frac{\tan \alpha}{1 + \tan^2 \alpha}$ is equal to

a) $\frac{9}{25}$

b) $\frac{12}{25}$

c) $\frac{3}{4}$

d) $\frac{1}{25}$

Answer (b)

Explanation:

$$\therefore \sec \alpha = \frac{5}{4}, \quad \therefore \tan \alpha = \sqrt{\sec^2 \alpha - 1}$$

$$= \sqrt{\frac{25}{16} - 1} = \sqrt{\frac{25 - 16}{16}} = \sqrt{\frac{9}{16}} = \frac{3}{4}$$

Now,

$$\frac{\tan \alpha}{1 + \tan^2 \alpha} = \frac{3/4}{1 + (3/4)^2} = \frac{3/4}{1 + 9/16} = \frac{3/4}{25/16} = \frac{12}{25}$$

2. If $\tan A = 1$ and $\tan B = \sqrt{3}$, then $\cos A \cdot \cos B \sin A \cdot \sin B$ is equal to

a) $\frac{1+\sqrt{3}}{2\sqrt{2}}$

b) $\frac{1-\sqrt{3}}{2\sqrt{2}}$

c) $\frac{2\sqrt{2}}{3}$

d) 1

Answer (b)

Explanation:

$$\tan A = 1,$$

$$\sin \frac{\tan A}{\sqrt{1+\tan^2 A}} = \frac{1}{\sqrt{2}}$$

$$\cos A = \frac{1}{\sqrt{1+\tan^2 A}} = \frac{1}{\sqrt{2}}$$

$$\tan B = \sqrt{3},$$

$$\sin B = \frac{\sqrt{3}}{\sqrt{1+3}} = \frac{\sqrt{3}}{2}$$

$$\cos B = \frac{1}{\sqrt{1+3}} = \frac{1}{2}$$

$$\therefore \cos A \cdot \cos B - \sin A \cdot \sin B$$

$$= \frac{1}{\sqrt{2}} \cdot \frac{1}{2} - \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{3}}{2} = \frac{1-\sqrt{3}}{2\sqrt{2}}$$

3. Given that $16 \cot \theta = 12$, then $\frac{\sin \theta + \cos \theta}{\sin \theta - \cos \theta}$ is equal to

a) 7

b) -7

c) $\frac{1}{7}$

d) $\frac{2}{7}$

Answer (a)

Explanation:

$$16 \cot \theta = 12 \Rightarrow \cot \theta = \frac{12}{16} = \frac{3}{4}$$

$$\therefore \frac{\sin \theta + \cos \theta}{\sin \theta - \cos \theta} = \frac{\frac{\sin \theta}{\sin \theta} + \frac{\cos \theta}{\sin \theta}}{\frac{\sin \theta}{\sin \theta} - \frac{\cos \theta}{\sin \theta}}$$

$$= \frac{1 + \cot \theta}{1 - \cot \theta} = \frac{1 + \frac{3}{4}}{1 - \frac{3}{4}} = \frac{7/4}{1/4} = 7$$

4. If $\tan \theta + \frac{1}{\tan \theta} = 2$ then the value of $\tan^2 \theta + \frac{1}{\tan^2 \theta}$ is equal to

- a) 6
- b) 4
- c) 2
- d) 3

Answer (c)

Explanation:

$$\tan \theta + \frac{1}{\tan \theta} = 2$$

Squaring on both sides,

$$\tan^2 \theta + \frac{1}{\tan^2 \theta} + 2 \cdot \tan \theta \cdot \frac{1}{\tan \theta} = 4$$

$$\Rightarrow \tan^2 \theta + \frac{1}{\tan^2 \theta} = 4 - 2 = 2$$

5. If $\tan \theta = \frac{12}{13}$, then $\frac{2 \sin \theta \cos \theta}{\cos^2 \theta - \sin^2 \theta}$ is equal to

a) $\frac{123}{25}$

b) $\frac{312}{25}$

c) $\frac{231}{25}$

d) $\frac{192}{25}$

Answer (b)

Explanation:

$$\tan \theta = \frac{12}{13}$$

$$\therefore \frac{2 \sin \theta \cos \theta}{\cos^2 \theta - \sin^2 \theta} = \frac{2 \tan \theta}{1 - \tan^2 \theta} = \frac{2 \times \frac{12}{13}}{1 - \frac{144}{169}}$$

$$= 2 \times \frac{12}{13} \times \frac{169}{25} = \frac{312}{25}$$

6. If $\sqrt{3} \tan 2\theta - 3 = 0$, then the angle θ is

- a) 30°
- b) 60°
- c) 90°
- d) None of these

Answer (a)

Explanation:

$$\sqrt{3} \tan 2\theta - 3 = 0$$

$$\Rightarrow \sqrt{3} \tan 2\theta = 3$$

$$\Rightarrow \tan 2\theta = \frac{3}{\sqrt{3}} = \tan 60^\circ$$

$$\Rightarrow 2\theta = 60^\circ$$

$$\Rightarrow \theta = 30^\circ$$

7. The value of $\frac{\tan^2 60^\circ + 4 \sin^2 45^\circ + 3 \sec^2 30^\circ + 5 \cos^2 90^\circ}{\csc 30^\circ + \sec 60^\circ - \cot^2 30^\circ}$

- a) 5
- b) 3
- c) 9
- d) 2

Answer (c)

Explanation:

$$\begin{aligned} & \frac{\tan^2 60^\circ + 4 \sin^2 45^\circ + 3 \sec^2 30^\circ + 5 \cos^2 90^\circ}{\csc 30^\circ + \sec 60^\circ - \cot^2 30^\circ} \\ &= \frac{(\sqrt{3})^2 + 4\left(\frac{1}{\sqrt{2}}\right)^2 + 3\left(\frac{2}{\sqrt{3}}\right)^2 + 5(0)}{2 + 2 - (\sqrt{3})^2} \\ &= \frac{3 + 2 + 4 + 0}{1} = 9 \end{aligned}$$

8. From the top of a pillar of height 20m the angles of elevation and depression of the top and bottom of another pillar are 30° and 45° respectively. The height of the second pillar (in metre) is:

a) $\frac{20}{\sqrt{3}}(\sqrt{3}-1)m$

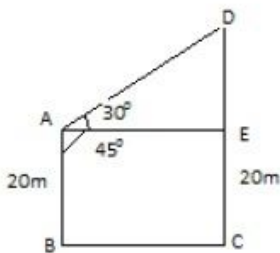
b) $\frac{20}{\sqrt{3}}(\sqrt{3}+1)m$

c) $20\sqrt{3}m$

d) $\frac{20}{\sqrt{3}}m$

Answer (b)

Explanation:



Let AB and CD are pillars.

Let DE = h

From $\triangle ADE$, $\tan 30^\circ$

$$= \frac{h}{AE} = \frac{1}{\sqrt{3}} \Rightarrow AE = h\sqrt{3} \quad \dots\dots(i)$$

From $\triangle ACE$,

$$= \tan 45^\circ = \frac{20}{AE} \Rightarrow AE = 20$$

From equation (i),

$$20 = h\sqrt{3} \Rightarrow h = \frac{20}{\sqrt{3}}m$$

$$\therefore \text{Required height} = 20 + \frac{20}{\sqrt{3}} = \frac{20}{\sqrt{3}}(\sqrt{3}+1)m$$

9. The Value of $(\tan 1^\circ \tan 2^\circ \tan 3^\circ \dots \tan 89^\circ)$ is:

- a) 1
- b) 89
- c) undefined
- d) 0

Answer (a)

Explanation:

$$\tan 1^\circ \cdot \tan 2^\circ \cdot \tan 3^\circ \dots \tan 89^\circ$$

$$\because \tan 89^\circ = \tan(90^\circ - 1^\circ)$$

$$= \cot 1^\circ$$

$$\tan 88^\circ = \tan(90^\circ - 2^\circ)$$

$$= \cot 2^\circ$$

$$\therefore \text{Expression} = (\tan 1^\circ \cdot \cot 1^\circ)$$

$$(\tan 2^\circ \cdot \cot 2^\circ) \dots \tan 45^\circ = 1$$

$$[\tan \theta \cdot \cot \theta = 1 \quad \tan(90^\circ - \theta) = \cot \theta]$$

10. If $\cos x + \cos^2 x = 1$, the numerical value of $(\sin^{12} x + 3\sin^{10} x + 3\sin^8 x + \sin^6 x - 1)$ is :

- a) -1
- b) 2
- c) 0
- d) 1

Answer (d)

Explanation:

$$\cos x + \cos^2 x = 1$$

$$\Rightarrow \cos x = 1 - \cos^2 x = \sin^2 x$$

$$\therefore \sin^{12} x + 3\sin^{10} x + 3\sin^8 x + \sin^6 x - 1$$

$$= (\sin^4 x + \sin^2 x)^3 - 1$$

$$= (\cos^2 x + \sin^2 x)^3 - 1$$

$$= 1 - 1 = 0$$

11. If $\operatorname{cosec} 2A = \sec\left(A - \frac{\pi}{4}\right)$, then $A =$

a) $\frac{4\pi}{3}$

b) $\frac{3\pi}{4}$

c) $\frac{\pi}{4}$

d) $\frac{3\pi}{2}$

Answer (c)

Explanation:

$$\operatorname{cosec} 2A = \sec\left(A - \frac{\pi}{4}\right)$$

$$\operatorname{cosec} 2A = \operatorname{cosec}\left(\frac{\pi}{2} - \left(A - \frac{\pi}{4}\right)\right)$$

$$\operatorname{cosec} 2A = \operatorname{cosec}\left(\frac{\pi}{2} - A + \frac{\pi}{4}\right)$$

$$\operatorname{cosec} 2A = \operatorname{cosec}\left(\frac{3\pi}{4} - A\right)$$

$$2A = \frac{3\pi}{4} - A$$

$$3A = \frac{3\pi}{4}$$

$$A = \frac{\pi}{4}$$

12. Evaluate: $\cos(40^\circ - \theta) - \sin(50^\circ + \theta) + \frac{\cos^2 40^\circ + \cos^2 50^\circ}{\sin^2 40^\circ + \sin^2 50^\circ}$

a) 1

b) -1

c) -2

d) 0

Answer (a)

Explanation:

$$\begin{aligned} & \cos(40^\circ - \theta) - \sin(50^\circ + \theta) + \frac{\cos^2 40^\circ + \cos^2 50^\circ}{\sin^2 40^\circ + \sin^2 50^\circ} \\ &= \sin\{90^\circ - (40^\circ - \theta)\} - \sin(50^\circ + \theta) \\ & \quad + \frac{\cos^2 40^\circ + \cos^2(90^\circ - 40^\circ)}{\sin^2 40^\circ + \sin^2(90^\circ - 40^\circ)} \left[\because \cos \alpha = \sin(90^\circ - \alpha) \right] \\ &= \sin(50^\circ + \theta) - \sin(50^\circ + \theta) + \frac{\cos^2 40^\circ + \sin^2 40^\circ}{\sin^2 40^\circ + \cos^2 40^\circ} \\ &= 0 + 1 = 1 \end{aligned}$$

13. Evaluate: $\frac{\cos 70^\circ}{\sin 20^\circ} + \frac{\cos 55^\circ \operatorname{cosec} 35^\circ}{\tan 5^\circ \tan 25^\circ \tan 45^\circ \tan 65^\circ \tan 85^\circ}$

- a) 1
- b) 2
- c) 3
- d) 4

Answer (b)

Explanation:

$$\begin{aligned} & \frac{\cos 70^\circ}{\sin 20^\circ} + \frac{\cos 55^\circ \operatorname{cosec} 35^\circ}{\tan 5^\circ \tan 25^\circ \tan 45^\circ \tan 65^\circ \tan 85^\circ} \\ &= \frac{\cos(90^\circ - 20^\circ)}{\sin 20^\circ} + \frac{\cos 55^\circ \operatorname{cosec}(90^\circ - 55^\circ)}{\tan 5^\circ \tan 25^\circ \tan 45^\circ} \\ & \qquad \qquad \qquad \tan(90^\circ - 25^\circ) \tan(90^\circ - 5^\circ) \end{aligned}$$

$$= \frac{\sin 20^\circ}{\sin 20^\circ} + \frac{\cos 55^\circ \operatorname{sec} 55^\circ}{\tan 5^\circ \tan 25^\circ \tan 45^\circ \cot 25^\circ \cot 5^\circ}$$

$$= 1 + \frac{1}{\tan 45^\circ} = 1 + 1 = 2$$

$$[\because \tan 5^\circ \cot 5^\circ = 1 \text{ and } \tan 25^\circ \cdot \cot 25^\circ = 1]$$

14. Evaluate: $2\left(\frac{\cos 58^\circ}{\sin 32^\circ}\right) - \sqrt{3}\left(\frac{\cos 38^\circ \operatorname{csc} 52^\circ}{\tan 15^\circ \tan 60^\circ \tan 75^\circ}\right)$

a) 1

b) 2

c) 3

d) 0

Answer (a)

Explanation:

$$\begin{aligned} & 2\left(\frac{\cos 58^\circ}{\sin 32^\circ}\right) - \sqrt{3}\left(\frac{\cos 38^\circ \operatorname{csc} 52^\circ}{\tan 15^\circ \tan 60^\circ \tan 75^\circ}\right) \\ &= 2\left\{\frac{\cos(90^\circ - 32^\circ)}{\sin 32^\circ}\right\} - \sqrt{3}\left\{\frac{\cos 38^\circ \operatorname{csc}(90^\circ - 38^\circ)}{\tan 15^\circ \tan 60^\circ \tan(90^\circ - 15^\circ)}\right\} \\ &= 2\left(\frac{\sin 32^\circ}{\sin 32^\circ}\right) - \sqrt{3}\left\{\frac{\cos 38^\circ \operatorname{sec} 38^\circ}{\tan 15^\circ \times \sqrt{3} \times \cot 15^\circ}\right\} \\ &= 2 - \sqrt{3}\left\{\frac{\cos 38^\circ \times \frac{1}{\cos 38^\circ}}{\tan 15^\circ \times \sqrt{3} \times \frac{1}{\tan 15^\circ}}\right\} \\ &= 2 - \frac{\sqrt{3}}{\sqrt{3}} = 2 - 1 = 1 \end{aligned}$$

15. Evaluate: $\cos 1^\circ \cdot \cos 2^\circ \cdot \cos 3^\circ \dots \cos 180^\circ$

a) 0

b) 2

c) 1

d) -1

Answer (a)

Explanation:

$$\cos 1^\circ \cdot \cos 2^\circ \cdot \cos 3^\circ \dots \cos 180^\circ$$

$$= \cos 1^\circ \cos 2^\circ \cos 3^\circ \dots \cos 89^\circ \cos 90^\circ \cos 91^\circ \dots \cos 180^\circ$$

$$= \cos 1^\circ \cos 2^\circ \cos 3^\circ \dots 0 \times \cos 91^\circ \dots \cos 180^\circ = 0$$

$$[\because \cos 90^\circ = 0]$$

16. If P, Q, R are the interior angles of a triangle PQR, then $\tan \frac{Q+R}{2} = ?$

a) $\cot P$

b) $\cot \frac{P}{2}$

c) $\tan P$

d) $\tan \frac{P}{2}$

Answer (b)

Explanation: In ΔPQR , we have

$$P + Q + R = 180^\circ$$

$$\Rightarrow Q + R = 180^\circ - P$$

$$\Rightarrow \frac{Q+R}{2} = 90^\circ - \frac{P}{2}$$

$$\Rightarrow \tan \left(\frac{Q+R}{2} \right) = \tan \left(90^\circ - \frac{P}{2} \right)$$

$$\Rightarrow \tan \left(\frac{Q+R}{2} \right) = \cot \frac{P}{2}$$

17. Evaluate: $(\sin \theta + \operatorname{cosec} \theta)^2 + (\cos \theta + \sec \theta)^2$

a) $7 + \tan^2 \theta$

b) $7 + \tan^2 \theta + \cot^2 \theta$

c) $5 + \tan^2 \theta + \cot^2 \theta$

d) $5 - \tan^2 \theta + \cot^2 \theta$

Answer (b)

Explanation: $(\sin \theta + \operatorname{cosec} \theta)^2 + (\cos \theta + \sec \theta)^2$

$$= (\sin^2 \theta + \operatorname{cosec}^2 \theta + 2 \sin \theta \operatorname{cosec} \theta) + (\cos^2 \theta + \sec^2 \theta + 2 \cos \theta \sec \theta)$$

$$= \left(\sin^2 \theta + \operatorname{cosec}^2 \theta + 2 \sin \theta \frac{1}{\sin \theta} \right)$$

$$= \left(\cos^2 \theta + \sec^2 \theta + 2 \cos \frac{1}{\cos \theta} \right)$$

$$= (\sin^2 \theta + \operatorname{cosec}^2 \theta + 2) + (\cos^2 \theta + \sec^2 \theta + 2)$$

$$= \sin^2 \theta + \cos^2 \theta + \operatorname{cosec}^2 \theta + \sec^2 \theta + 4$$

$$= 1 + (1 + \cot^2 \theta) + (1 + \tan^2 \theta) + 4$$

$$\left[\because \operatorname{cosec}^2 \theta = 1 + \cot^2 \theta, \sec^2 \theta = 1 + \tan^2 \theta \right]$$

$$7 + \tan^2 \theta + \cot^2 \theta$$





20. Evaluate: $\frac{\tan \theta + \sin \theta}{\tan \theta - \sin \theta}$

a) $\frac{\sec \theta + 1}{2}$

b) $\frac{\sec \theta - 1}{\sec \theta + 1}$

c) $\frac{\sec \theta + 1}{\sec \theta - 1}$

d) $\frac{2 \sec \theta}{\sec \theta + 1}$

Answer (c)

Explanation: $\frac{\tan \theta + \sin \theta}{\tan \theta - \sin \theta}$

$$\begin{aligned} &= \frac{\frac{\sin \theta}{\cos \theta} + \sin \theta}{\frac{\sin \theta}{\cos \theta} - \sin \theta} = \frac{\sin \theta \left(\frac{1}{\cos \theta} + 1 \right)}{\sin \theta \left(\frac{1}{\cos \theta} - 1 \right)} \\ &= \frac{\frac{1}{\cos \theta} + 1}{\frac{1}{\cos \theta} - 1} = \frac{\sec \theta + 1}{\sec \theta - 1} \end{aligned}$$